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Optimizing Resource Utilization Using an Assignment Model in Case of Metal Workshop Operation

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Abstract

Optimal resource utilization is the critical measuring parameter in industrial operational activities like assigning numerous workers to machines. The Assignment Problem (AP) is one of the classical linear methodological models that is categorized as a special form of transportation problem in linear programming procedurally implemented through the Hungarian algorithm. The case firm and the other related stakeholders are still facing challenges and difficulties in making factual decisions in their industrial operational activities, special in resources assigning, scheduling, and monitoring the progress of their works. Former scholars tried to advance to address the tricks. But, still, there are gaps especially in terms of linking these roles of assignment algorithms with real cases. This paper presented the procedure, necessity, and aim of the Hungarian Algorithmic method. This article aims to analyze assignment activities. After the parameters were identified, the unbalanced matrix of the real case of Metal Workshop was converted into a balanced (square matrix) by adding sufficient imaginary workers or machines. Via this algorithm, the optimal cost per unit was indicated as 18, and the optimized profit from the assignment was indicated as 100. Having the findings and deployed approach, the article forwarded lists of recommendations to ensure the long-run competitiveness of the company.

Keywords: Linear programming, Hungarian Algorithm, Assignment problem, Resource utilization, Optimization.

1 | Introduction

In business operational concerns, the optimal assignment of limited resources is a very critical managerial activity [1]–[4]. It required holistic and basement flows to conduct factual decisions. According to Operational Researcher Quarterly (1962), [5]–[8], an assignment model is typical of a transportation problem where the objective is to minimize the cost of allocating several jobs to several persons or facilities so that one person or facility is assigned to only one job. The classical solution to the AP given is the Hungarian Algorithmic Approach (HAA), originated by Kuhn in 1955 [9]–[11]. Broadly, it is also an element of the linear programming model that emphasizes the problems of assigning each facility to one and only one job to

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optimize efficiency [12]–[14]. Its structure follows the transportation problem model rather than supply at sources and demand at the destination is limited to a single entity.

The purpose of this classical linear methodological model is to ensure that the activities of management operations, along with a scientific quantitative management approach, that expected to be used to determine the minimum cost of assignment of workers to specific jobs or certain machines to workstations through obeying optimal operational scenarios. Optimality can be achieved when total cost is reduced and overall profit could be enhanced [4], [15]–[17]. On the ground, in numerous industrial operational activities, managers phase critical decision points that expose them to seeking an optimal assignment of 'n' facility to 'j' jobs [6], [18]–[20]. Majorly in the process of scheduling the operational activities, an optimum resource should be assigned, to achieve the pre-defined organizational ultimate goal.

The current case is also unable to deploy and optimize the assignment operation and is obliged to incur quarterly 10% beyond its pre-budgeted operational expenses. This is due to the inability to tackle the trick of assigning the "i" operator to the "M/c" machine optimally using quantitative management techniques. In most former studies, the assignment model is used by emphasizing mathematical computations for both balanced and unbalanced models [14], [21]–[23]. For instance, lists of scholars have advanced several techniques to address generalized APs in real cases [7], [24]–[26]. An unbalanced matrix is converted to a balanced form or square matrix by adding sufficient numerals of the dummy (imaginary worker or machine) through emphasizing the problem analysts need to face, and all costs for these added new columns or rows are zero where do not go to change the pre-defined objective function.

In reality, these unbalanced resources should be considered, and operational management is expected to make all resource utilization feasible and competitive. This is what the current paper is interlinked to the HAA. In this case metal workshop, there are about 7 workstations and 8 operators in the production processes with their corresponding cost matrix and potential entities, which are depicted in *Table 3*.

This article aims to analyze the AP of the above-mentioned case scale manufacturing industry using the assignment model, specifically, Hungarian Algorithmic methodology to optimize such as minimize distance and maximize interaction of entities in resources assignments operations within a facility. The study framework is framed in *Fig. 1* below. The specific objective that this article deployed with procedural tasks of Hungarian methods such as row operation, column operation, and row and column scanning for optimizing the resource allocation operation. The significance of this article is enabling the predominantly current case metal shop to allocate its resources optimally. Secondly, the rationale of this study is to facilitate the practices and implementation scenarios of the quantitative management approach by applying a linear programming model in a small-scale manufacturing operation.

Quantitative Management Window (QM for Window) was used as an analysis tool. Of course, there are a lot of analysis tools. However, this Software Package is suitable and easy to deploy for industrial practitioners and management of daily operational activities. The result was discussed and interpreted along with the aim of this study and generic business operational activities. To actualize this tool and enable the relevant enterprises, the author enumerated some recommendations for the long-run optimal resource allocation in real cases.

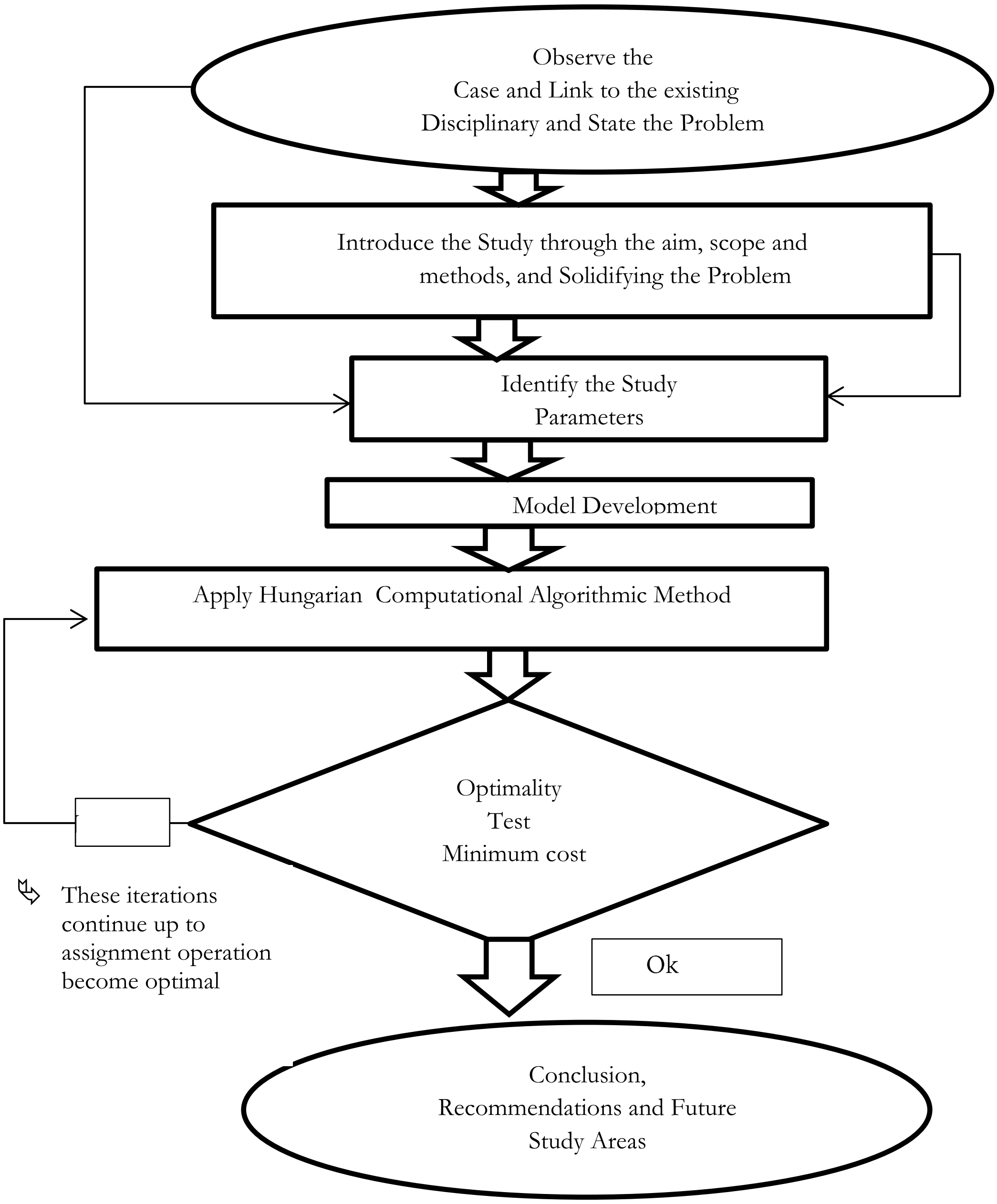

Fig. 1. Study framework.

2 | Literature Review

2.1 | Theoretical Background of Assignment Problem

Since the Industrial Revolution, mathematical formulation of real-world operational scenarios has been advanced, and competitiveness in the limited resources has been critical managerial acuities [19], [27]–[29]. Scholars and industrial practitioners figured out this as optimal resource utilization [2], [7], [30]. Resources utilization is when an industrial operational activity is realistically monitored and confirmed by the fundamental concepts of quantitative decision-making scenarios [4], [31]. Scholars and former researchers built theoretical and conceptual approaches such as APs [5], [28], [32]. AP is the classical approach of optimization algorithm and special structure of the Transportation Problem mathematical model [6], [10], [33].

The pre-conditions for deploying an AP are the cost matrix table needs to be balanced or square matrixes unless Dummy job/activities and workers/machine should be added with entries such that supply and demand are equal. According to the classical approach of the Linear Programming (LP) model, there are types of algorithms in the process of evaluation of AP in quantitative management operations [10], [34]. The mathematical model of AP is structured as a Transportation problem except that the supply is in each supplier and the demand in each destination is one which is mentioned as depicted in *Table 1* of the methodology section.

2.2 | Types of Assignment Algorithms

Algorithms are composed of a set of instructions that are usually followed in defined step-by step procedural activities [17], [26], [35]. The objective of these algorithms in LP is to produce a feasible solution to a defined model to find an optimal solution. Scholars classified algorithms in two main classifications: Heuristic Algorithms and classical Algorithms [36]–[38].

Table. Algorithm in AP.

Algorithm Heuristic	Classical Algorithm
Greedy algorithm	Hungarian Algorithm
Genetic algorithm	Simplex method
Penalty algorithm	Brute force algorithm
Harmony algorithm	Branched algorithm

The goal of these algorithms is minimizing an opportunity loss or cost (C_{ij}) in the process of assigning i^{th} resources (worker or machine) and j^{th} task (job) such that i=1, 2, 3…n and j=1, 2, 3…n, and Xij =1, when worker i is assigned to job j, Xij =0, otherwise.

$$\text{Gross Minimization Cost } (Z) = \sum_{i=1}^{n} \sum_{j=1}^{n} \text{XijCij}.$$

Previous studies suggested that the Hungarian Algorithm, which was founded by two mathematicians, D. Konig and E. Egervary, was a suitable computational algorithm in AP for ordinary operational areas[3], [10], [23]. An iteration of this algorithm is evaluated step by step through an assuring optimality test [26], [39].

Works of literature indicate that mathematical formulation of the real world has been solved through different approaches based on various aspects (deployment cost, suitability, running time, relevance of the input parameters, and interest of the researcher). The current study agrees with this overall and fundamental concept and tried further interlinking the necessity and potential implication of applying AP in real cases.

3 | Methodology

3.1 | Model Formulation

In the process of analysis and evaluations of optimal resource allocation in real cases, formulating the model using the key input parameter is the crucial phase to meet the goal of the activities. Having these fundamental concepts, researchers have been internalizing a lot of scholarly evaluated analysis. For instance, the ‘n’ facility should be assigned to the ‘j’ job through the assignment possibilities of ‘n!’ in the AP-solving model. For instance, the ‘n’ facility should be assigned to the ‘j’ job through the assignment possibilities of ‘n!’ in the AP-solving model [11], [13], [31], [40], stated that the simplest way of seeking an optimized assignment procedure is by enumerating all “n!” possible arrangements and evaluate their corresponding total cost and select the resources assignment with minimum cost. Structurally, an AP is similar to the transportation model, as depicted in the following *Table 2*.

Table 2. Generic form of transportation model table.

	Activities (Jobs)				
	1	1	…	n	
1	C_{11}	C_{12}	…	C_{1n}	1
1	C_{21}	C_{22}	… .	C_{2n}	1
	.				.
Resources (operators)	.				.
	.				.
n	C_{n1}	C_{n2}	…	C_{nn}	1
	1	1	…	1	

The element C_{ij} (space, time, energy, etc.) represents the measurement of effectiveness when the i^{th} operator is assigned to j^{th} job in resource allocations. The element X_{ij} represents the number of i^{th} individuals assigned to the j^{th} job by following the principle of assignment model that follow i^{th} operator needs to be assigned to only 1 job and jth job also assigned to 1 operator can be guided by the following structure.

This means it is also assumed that $X_{ij}=1$, if "i", the resources (workers) are assigned to "j", activities (jobs) $X_{ij}=0$, otherwise.

3.2 | Classical Mathematical Formulation for Assignment Problem

Having the parameter in the above generic matrix table, the mathematical model formulation for the AP can be stated as follows.

Define the activity variables:

$$\text{Xij} = \begin{cases} 1, & \text{if workers i is assigned to job j,} \\ 0, & \text{otherwise.} \end{cases}$$

Then, the AP can be formulated mathematically as follows:

Minimize

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} \text{XijCij},$$

Subject to

$$\sum_{j=1}^{n} \text{Xij} = 1 \text{ for all i (availability).}$$

i.e., each worker should be assigned to one and only one job. Which is implied that, $X_{i1}+X_{i2}+X_{i3}...+X_{in}=1$, i=1, 2, 3…n.

$$\sum_{i=1}^{n} \text{Xij} = 1 \text{ for all j (requirement).}$$

i.e., each worker should be assigned to one and only one job. Which is implied that, $X_{1j}+X_{2j}+X_{3j}...+X_{nj}=1$, j=1, 2, 3…n.

$X_{ij}=0$, otherwise.

The pre-conditions for applying this mathematical model of AP are two: 1) the cost matrix should be a square matrix, and 2) an optimal solution table (matrix) for the problem would have only one assignment in a given row or column.

Any basic feasible solution of an AP contains 2n-1 variables that contain n-1 zero variables [12]–[14], [24], [28], [41]. This high degree of degeneracy makes computational procedures of transportation problems inefficient. [15], [31] underscored that to overcome these computational inconveniences, the separation method is a feasible technique for APs conveniently by applying the HAA. The current study acknowledged the fundamental concepts of APs and tried to interlink the potential application of the algorithmic approach (Hungarian computation method). This is the critical managerial issue to make decisions quantitatively.

3.3 | Hungarian Algorithmic Approach

It is a series of computational steps in solving APs in a linear programming model for both maximization and minimization objective functions [2], [18], [23], [31], [42]. This study majorly emphasized optimizing resource allocation through minimization which can affect optimal resource utilization and boost overall efficiencies of operational activities.

Step 1. Based on the provided real case problem, facilitate the cost table. Technically, analysts need to check whether the AP is either balanced or not. If the number of origins is not equal to the number of destinations, dummy origins or destinations must be added.

Step 2. Apply row operation by finding out the smallest cost cell, and subtracting this smallest cost element from each cell of that row of the cost table. This can make the table have at least one zero in each row of the cost table. This new table is known as the reduced-cost table.

Step 3. Apply column operation by finding out the smallest cost element in each column and reduce it from each cell of that column cost table. By this step, each row and column has at least one zero element, which is known as the second reduced cost table.

Step 4. At this step, determine an assignment through the following:

I. For each row or column with a single zero element that has not been assigned or counselled, the box that zero cells as an assigned element cell.

II. For every zero that becomes assigned, cross out all other zeros in the same column and for rows.

III. If a row and column there are 2 or more zeros and one can't be chosen through steps, choose the assigned zero arbitrarily.

IV. The above procedures may be repeated till every zero cell is boxed (assigned) or crossed out (counselled out).

Step 5. Through scanning the newly generated cost/profitability, an assignment is found if the number of assigned cells is equal to the number of rows (and columns). In a case of arbitrarily chosen, there may be an alternative optimum solution is there or identified. At this phase/step/ if there is no optimum solution is obtained, and then proceed to *Step 6*.

Step 6. In this step, the analyst needs to draw a set of lines equal to the number of assignments which has been made in *Step 4*, covering all the zeros using the following examination.

I. Marking check (☑) to those rows where No assignment has been addressed. Examine the checked (☑) rows.

II. If any zero cell occurs in those rows, check (☑) the respective columns that contain those particular zeros.

III. Examine the checked (☑) columns. If any assigned zero element occurs in those columns, check (☑) the respective rows that contain those assigned zeros.

The process may be iterated till now more rows or columns can be checked. Again, draw lines through all unchecked rows and rough all checked columns.

Step 7. At this step, activities of examining those elements that are not covered by lines must be conducted. Then, choose the smallest of these cost elements and subtract this smallest one from all the elements that do not have a line through them. And, add this smallest to every element that lies at the intersection of two lines.

As a result, the new revised profit table can be generated. So, these steps of the Hungarian Algorithmic computational methods were used to achieve the objective of the study.

4 | Analysis and Discussions

The input parameters of this AP are extracted from the case enterprise's operational management data. The relevant parameters required for the deployment of the pre-defined algorithmic computational method are cost tables for minimizing assignment cost. This type of cost is the expenses incurred for allocating or moving operators to the specific workstation, as depicted in *Table 3* below. Also, this article is only intended to utilize the provided data in scientific activities without exposing other internal business-secured data. The aim is to make the small and medium-scale business entrants with a quantitative management approach that is essential in the decision-making process.

Table 3. Data (cost matrix) in assignment operations of operator (i) to job (j).

		Jobs						
		1	2	3	4	5	6	7
Operator	A	16	7	6	9	12	5	16
	B	10	9	4	5	6	11	2
	C	2	8	5	6	8	14	7
	D	9	5	6	7	14	2	9
	E	7	11	19	13	9	15	6
	F	5	3	11	15	13	6	8
	G	2	12	5	8	3	14	10
	H	12	15	8	10	3	5	9

4.1 | Analysis and Discussion

The analysis was conducted separately for both cost minimization and profit maximization using input parameters of the assignment model that were already put in *Table 3*.

Linear programming problem formulation

In this article, two objective functions are internalized to address the pre-defined goal of the study. In the linear programming problem, these objective functions (Z) need to be formulated through the principle of the generic mathematical formulation of the linear programming problem that is depicted in the previous *Fig. 1*. These familiar typical objective functions are as below:

Minimization

– *Minimizing the cost incurred due to moving an operator (i) to job (j).*

Maximization

– *Maximizing profit of the assignment of worker or operator (i) to job (j).*

Having this aim, procedurally, an author analyzed the optimal AP of this article in the following sections.

Minimization

Since an AP is a special form of linear programming problem; the above cost matrix can be converted into a mathematical model for the minimization objective as the following.

4.2 | Decision Variable

Based on its computational capability, Hungarian Algorithmic methodology is preferred to solve the above AP mathematical model optimally. The former steps enumerated one up to six, deployed as the following through different iterations of the Algorithm.

The number of operators (i) is not equal to the number of jobs (j), which is not a Balanced Assignment Problem (BAP). Before applying computational iterations, the Dummy Job with 'zero cost' should be added, and the unbalanced problem should be converted to a balanced structure as follows. This is step one of Hungarian computational methods.

X_{ij} = 1 if the operator (i) is assigned to Job (j) = 0, otherwise.

Where: i = operator A, B, C, D, E, F, G, H and **j**= Job 1, 2, 3, 4, 5, 6, 7 respectively.

Minimization (Z) =
$16X_{11}+7X_{12}+6X_{13}+9X_{14}+12X_{15}+5X_{16}+16X_{17}+10X_{21}+9X_{22}+4X_{23}+5X_{24}+6X_{25}+11X_{26}+2X_{27}+2X_{31}+8X_{32}+5X_{33}+6X_{34}+8X_{35}+14X_{36}+7X_{37}+9X_{41}+5X_{42}+6X_{43}+7X_{44}+14X_{45}+2X_{46}+9X_{47}+7X_{51}+11X_{52}+19X_{53}+13X_{54}+9X_{55}+15X_{56}+6X_{57}+5X_{61}+3X_{62}+11X_{63}+15X_{64}+13X_{65}+6X_{66}+8X_{67}+2X_{71}+12X_{72}+5X_{73}+8X_{74}+3X_{75}+14X_{76}+10X_{77}+12X_{81}+15X_{82}+8X_{83}+10X_{84}+3X_{85}+5X_{86}+9X_{87}.$

Subject to:

$X_{11}+X_{12}+X_{13}+X_{14}+X_{15}+X_{16}+X_{17}+X_{18}=1$, no more than one operator (i) assigned to job (j).

$X_{21}+X_{22}+X_{23}+X_{24}+X_{25}+X_{26}+X_{27}+X_{28}=1.$

$X_{31}+X_{32}+X_{33}+X_{34}+X_{35}+X_{36}+X_{37}+X_{38}=1.$

$X_{41}+X_{42}+X_{43}+X_{44}+X_{45}+X_{46}+X_{47}+X_{48}=1.$

$X_{51}+X_{52}+X_{53}+X_{54}+X_{55}+X_{56}+X_{57}+X_{58}=1.$

$X_{61}+X_{62}+X_{63}+X_{64}+X_{65}+X_{66}+X_{67}+X_{68}=1.$

$X_{71}+X_{72}+X_{73}+X_{74}+X_{75}+X_{76}+X_{77}+X_{78}=1.$

$X_{81}+X_{82}+X_{83}+X_{84}+X_{85}+X_{86}+X_{87}+X_{88}=1.$

$X_{11}+X_{12}+X_{13}+X_{14}+X_{15}+X_{16}+X_{17}+X_{18}=1$, no more than one Job (j) assigned to the operator (i)

$X_{21}+X_{22}+X_{23}+X_{24}+X_{25}+X_{26}+X_{27}+X_{28}=1.$

$X_{31}+X_{32}+X_{33}+X_{34}+X_{35}+X_{36}+X_{37}+X_{38}=1.$

$X_{41}+X_{42}+X_{43}+X_{44}+X_{45}+X_{46}+X_{47}+X_{48}=1.$

$X_{51}+X_{52}+X_{53}+X_{54}+X_{55}+X_{56}+X_{57}+X_{58}=1.$

$X_{61}+X_{62}+X_{63}+X_{64}+X_{65}+X_{66}+X_{67}+X_{68}=1.$

$X_{71}+X_{72}+X_{73}+X_{74}+X_{75}+X_{76}+X_{77}+X_{78}=1.$

$X_{81}+X_{82}+X_{83}+X_{84}+X_{85}+X_{86}+X_{87}+X_{88}=1.$

Constraints:

All $X_{ij}>=0$, and entries of i and j must be none-negative.

Above subject to part can be written by matrix transpose style, but an author selected this format for simplicity and clarity inclusively (all industry practitioners) implementing in real cases.

Table 4. Cost–matrix table after Step 1 Hungarian method applied to balance the problem.

		Jobs							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**Dummy**
	A	16	7	6	9	12	5	16	0
	B	10	9	4	5	6	11	2	0
	C	2	8	5	6	8	14	7	0
	D	9	5	6	7	14	2	9	0
Operator	E	7	11	19	13	9	15	6	0
	F	5	3	11	15	13	6	8	0
	G	2	12	5	8	3	14	10	0
	H	12	15	8	10	3	5	9	0

Now, the problem is balanced and possible to apply the computational algorithm's *Step 2,* which is enumerated in the above procedural section. According to this step, row operation must be applied through finding out the smallest cost element, and subtracting this smallest cost element from each cost element (cell) of the rows.

The aim is to make the table have at least one zero in each row of the reduced-cost table. Since all rows have at least zero cost cells, step two Hungarian method (row operation) has no effects on this converted cost table. That means the 1st reduced cost table can be as it is, which is already displayed in *Table 5* below.

Table 5. 1st reduced cost –matrix table after step two Hungarian algorithmic method.

		Jobs								
		1	**2**	**3**	**4**	**5**	**6**	**7**	**Dummy**	**Min-Row**
	A	16	7	6	9	12	5	16	0	0
	B	10	9	4	5	6	11	2	0	0
	C	2	8	5	6	8	14	7	0	0
	D	9	5	6	7	14	2	9	0	0
Operator	E	7	11	19	13	9	15	6	0	0
	F	5	3	11	15	13	6	8	0	0
	G	2	12	5	8	3	14	10	0	0
	H	12	15	8	10	3	5	9	0	0
	Min-column	2	3	4	5	3	2	2	0	

Step 3 (Hungarian Algorithmic Method). Here is the column operation. It is through finding out the smallest cost element in each column and reducing it from each cell of that corresponding column. As a result, each row and column is expected to have at least one zero cost element, which is known as the second reduced cost–matrix table. As the above table depicts, the smallest cost cell elements for all columns are 2, 3, 4, 5, 3, 2, 2, and 0; for columns of the above *Table 6,* are 1, 2, 3, 4, 5, 6, 7, and 8 respectively. The reduced cost-matrix table for the column operation using Hungarian Computational Iteration is depicted as the below *Table 5* which is considered as the 2nd reduced cost table HAA.

Table 6. 2nd reduced cost-matrix table Hungarian Algorithmic methods.

		Jobs								
		1	**2**	**3**	**4**	**5**	**6**	**7**	**Dummy**	**Min-Row**
	A	14	4	2	4	9	3	14	**0**	0
	B	8	6	**0**		3	9	0	0	0
	C	**0**	5	1	1	5	12	5	0	0
	D	7	2	2	2	11	**0**	7	0	0
Operator	E	5	8	15	7	6	13	4	0	0
	F	3	**0**	7	10	10	4	6	0	0
	G		9	1	3		12	8	0	0
	H	10	12	4	5	**0**	3	7	0	0
	Min-column	2	3	4	5	3	2	2	0	

Optimality test the assignment processes from the above *Step 3* Hungarian Algorithmic procedure. Since only 6 cost cell elements are covered, the assignment is not optimized. Now, the computational process proceeds to *Step 6* Hungarian method, which is elaborated in the above lists of its steps. In this step, the analyst needs to draw a set of lines equal to the number of assignments which has been made in *Step 4*; covering all the zeros examined iteratively up to the assignment is optimized.

Table 7. 3rd reduced cost-matrix table Hungarian Algorithmic method.

		Jobs							
		1	2	3	4	5	6	7	Dummy
Operator	A	14	4	2	4	9	3	14	0
	B	8	6	0	0	3	9	0	0
	C	0	5	1	1	5	12	5	0
	D	7	2	2	2	11	0	7	0
	E	5	8	15	7	6	13	4	0
	F	3	0	7	10	10	4	6	0
	G	0	9	1	3	0	12	8	0
	H	10	12	4	5	0	3	7	0
	K=1								

In this iteration, select the uncovered smallest cost cell from the reduced cost matrix and say it in any letter. For the current iteration, the author used the uncovered smallest cell as "k", which is 1 (k=1), and subtracted "k" from all uncovered cost elements to add it to the cost cell on which two zeros-covered lines intersect each other. Through this, the following reduced Cost–Cost-matrix table was generated.

Table 8. 4th reduced cost-matrix table Hungarian Algorithmic method.

		Jobs							
		1	2	3	4	5	6	7	Dummy
Operator	A	14	4	1	3	9	2	14	0
	B	9	6	0	0	4	9	0	1
	C	0	4	0	0	5	11	4	0
	D	8	2	2	2	12	0	7	1
	E	5	7	14	6	6	12	3	0
	F	4	0	7	10	11	4	6	1
	G	0	8	0	2	0	11	7	0
	H	10	11	3	4	0	2	6	0
	K=1								

Then, test the optimality by refreshing guidelines through *Steps 3* to *6* those mentioned earlier. The assignment operation is not optimized at this iteration because the number of covered by zero cost cells is not equal to the required number of assignments. Then, repeat the above iteration until all workers are assigned to the optimum workstation. As the above-reduced cost-matrix table depicts, the uncovered smallest cost cell is "1" or (k=1). Subtract k from all uncovered cost cells and add k to the intersection cost cell of two zero-covered lines. The Cost-Matrix of this step is reduced in the following *Table 9*.

Table 9. 5th reduced cost-matrix table Hungarian Algorithmic method.

		Jobs							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Operator	A	13	4	**0**	2	9	2	13	$\cancel{0}$
	B	9	7	$\cancel{0}$	$\cancel{0}$	5	10	**0**	2
	C	$\cancel{0}$	5	$\cancel{0}$	**0**	6	12	4	1
	D	7	2	1	1	12	**0**	6	1
	E	4	7	13	5	6	12	2	Unassigned
	F	3	**0**	6	9	11	4	5	1
	G	**0**	9	$\cancel{0}$	2	1	12	7	1
	H	9	11	2	3	**0**	2	5	$\cancel{0}$
	K=1								

Optimality test: as the above final Hungarian Algorithm iteration indicates, the assignment of workers to jobs is optimally assigned. The numbers of covered cost cells in this reduced cost matrix table are equal to the required numbers of assignments when the test is assured that the concept of allocation of resources through reducing loss of opportunities and enhancing feasibility, the iteration of the Hungarian computational algorithm.

4.3 | Final Assignment of Minimum Objective Function

Since the goal of the AP is minimizing the costs of allocating resources, the above analysis resulted or summarized as the following table of the final assignment of operator (i) to job (j). As mentioned in the previous section and study framework *Fig. 1*, optimizing is when activities are managed with minimum costs at which an optimal profit can be obtained.

Table 10. Minimum cost-based assignment of operator to job using Hungarian Algorithmic method.

Minimization Objective-based Assignment		
Operators (i)	**Jobs (j)**	**Corresponded Cost (C_{ij})**
A	job 3	Cost (C_{13}) = 6
B	job 7	Cost (C_{27}) = 2
C	job 4	Cost (C_{31}) = 2
D	job 6	Cost (C_{46}) =2
E	Job 5	Cost (C_{58}) =0
F	job 2	Cost (C_{62}) =3
G	job 1	Cost (C_{75}) =3
H	Unassigned (Dummy Job)	Cost (C_{84}) =0
	Total cost	18 units

Assignment*:* The network representation of the final optimal assignment depicted in *Table 10* of this real case quantitative operational management is depicted below in *Fig. 2*.

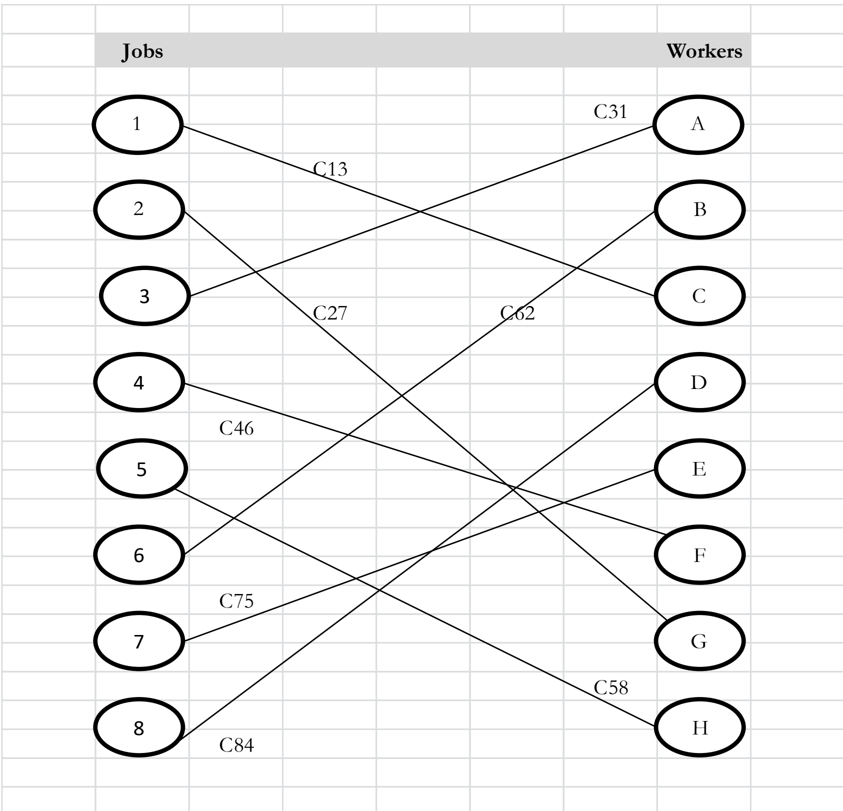

Fig. 2. Network representation of the optimum assignment of workers to jobs.

The total optimum assignment cost of this special form of transportation problem is computed by referencing the initial cost matrix that is displayed in *Table 4* of the *Step 1* Hungarian Algorithmic method.

Minimum assignment cost = $C_{13} + C_{27} + C_{34} + C_{46} + C_{58} + C_{62} + C_{71} + C_{85}$ = 6+2+2+2+0+3+2+3 = 18 per units

In resource allocation operations, the minimum cost of assigning an operator (i) to job (j) must be less than or equal to 18 units. To be a competitive and factual-driven decision-making approach, any business operational decision conducted in the current case manufacturing process should not compromise this optimum quantitative decision parameter.

Maximization

I. Maximization is the other objective function that is analyzed in this AP article. The aim is to enhance the profit in the resources assignment (assigning operator 'i' to job j) operations. The input parameter is the same as the cost-minimization function, which is already mentioned in the previous section, particularly in *Table 2,* which is again the input parameter for analyzing the maximizing operation in an AP.

The same is true for the maximization function in APs, which can be computed using a generic model of transportation problems in Linear Programming Problems (LPP). As elaborated in the minimization function, the Hungarian computational, algorithmic method (by going through *Steps* 1 up to 6 of the previous section) is preferable for maximizing an AP.

The procedures that have been used in the above minimization function are reused under this also. However, the difference is only selecting the maximum cell element from the Profit-Matrix while minimization is selecting the minimum cost element that should be selected with corresponding columns and rows. By applying the procedural Hungarian Algorithmic method, the final resources assignment (operator 'i' to job 'j') operations for maximizing profit were summarized as the following Step by Step.

4.4 | Hungarian Method for Maximization of an Assignment Problem

Step 1. It is balancing the matrix by adding dummy job (j) or Operator (i). It was already achieved by the previous minimization objective function by adding the dummy Job that is depicted in *Table 3* of the previous analysis.

Step 2 and Step 3. The Hungarian method for maximizing profit in the assignment of resources in the case: under this, row and column operations must be done. As mentioned earlier in the minimization objective, since all rows have at least one zero cell, the row operation of the Hungarian method does not affect the iteration. Again, also it is similar to the 1st Reduced Cost–Matrix Table after *Step 2* Hungarian Algorithmic method in the minimization objective function of the previous section. So, no need to duplicate the table, and the reader can refer the *Step 2* or the row operation from the incited table above.

The point here is the procedural differences between maximizing profit and minimizing distances in APs using the Hungarian Algorithm is the least cell element should be selected along with row (for row operation) and column (for column operation), while in maximization objective function follow the reverse which is the largest cell element should be selected from along with rows (for row operation) and columns (for column operation). Based on these pre-defined procedural approaches, the profit matrix is reduced to the following Table 8 after applying row and column operations referring to *Step 2*, *Step 3*.

Table 10. 2nd reduced profit-matrix table of Hungarian Algorithmic method.

		Jobs								
		1	2	3	4	5	6	7	Dummy	Min-Row
	A	0	-9	-13	-6	-2	-10	0	0	0
	B	-6	-6	-15	-10	-8	-4	-14	0	0
	C	-14	-7	-14	-9	-6	-1	-9	0	0
	D	-7	-10	-13	-8	0	-13	-7	0	0
Operator	E	-9	-4	0	-2	-5	0	-10	0	0
	F	-11	-12	-8	0	-1	-9	-8	0	0
	G	-14	-3	-14	-7	-11	-1	-6	0	0
	H	-4	0	-11	-5	-11	-10	-7	0	0
Max-Column		16	15	19	15	14	15	16	0	

Optimality test

In this phase, Hungarian Algorithmic computational analysis is evaluated based on the pre-defined procedural steps. The aim is to check if the order is equal to the supply entry in the cost matrix table.

Table 11. Optimality test for profit-matrix table of Hungarian Algorithmic method.

		Jobs							
		1	2	3	4	5	6	7	Dummy
	A	0	-9	-13	-6	-2	-10	0	0
	B	-6	-6	-15	-10	-8	-4	-14	0
	C	-14	-7	-14	-9	-6	-1	-9	0
	D	-7	-10	-13	-8	0	-13	-7	0
Operator	E	-9	-4	0	-2	5	0	-10	0
	F	-11	-12	-8	0	-1	-9	-8	0
	G	-14	-3	-14	-7	-11	-1	-6	0
	H	-4	0	-11	-5	-11	-10	-7	0

Since the number of zero-covered lines in the profit matrix is less than the required number, the AP has not achieved its maximum iteration. So, the author is indebted to proceed to the next step of the Hungarian algorithmic method that was procedurally deployed in the previous minimization objective function assignment. After lists of iterations, the algorithm achieved the maximum profit level in the assignment of operator (i) to job (j), as depicted in the following assignment.

Table 12. Maximized profit-matrix table of Hungarian Algorithmic method.

		Jobs							
		1	2	3	4	5	6	7	Dummy
	A	0	13	14	11	5	14	0	5
	B	0	4	10	9	6	3	8	0
	C	8	5	9	8	4	0	3	0
	D	3	10	9	9	0	14	3	2
Operator	E	9	9	0	5	8	4	9	5
	F	6	11	4	0	0	8	3	1
	G	8	1	9	6	9	1	0	2
	H	0	0	8	6	11	11	3	0
Max-Column		16	15	19	15	14	15	16	0

Optimality test

The lines of zero covered of cell element in the matrix equalized with the required numbers of an assignment. So, the Hungarian iteration could be stopped here and the assignment of profit maximization operation is achieved. This is an essential decision-making approach that all relevant industrial operational manager practitioners expect to realize these methods.

4.5 | Final Assignment of Maximum Objective Function

Since the goal of the AP is minimizing the costs of allocating resources, the above analysis resulted in the following table of the final assignment of the operator (i) to job (j). As mentioned in the previous section and study framework *Fig. 1*, optimizing is when activities are managed with minimum costs at which an optimal profit can be generated.

Table 13. Maximum profit-based assignment of operator (i) to job (j) using the Hungarian Algorithmic method.

	Maximization Objective-based Assignment								
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**Corresponded Cost (C_{ij})**
Job 1	16	7	6	9	12	5	Assign 16	0	Profit (P_{17}) = 16
Job 2	Assign 10	9	4	5	6	11	2	0	Profit (P_{21}) = 10
Job 3	2	8	5	6	8	Assign 14	7	0	Profit (P_{36}) = 14
Job 4	9	5	6	7	Assign 14	2	9	0	Profit (P_{45}) = 14
Job 5	7	11	Assign 19	13	9	15	6	0	Profit (P_{53}) =19
Job 6	5	3	11	Assign 15	13	6	8	0	Profit (P_{64}) =15
Job 7	2	Assign 12	5	8	3	14	10	0	Profit (P_{72}) =12
Job8 (Dummy)	0	0	0	0	0	0	0	Assign 0	Profit (P_{88}) =0
	Total profits or values from an assignment operations								100 Values

As the above final maximization of this current case firm's resources assignment in the operational management activities indicated, the optimal profit is shown as 100 values. To be commutative in the market arena, the resources must be optimally assigned and it must be greater than this optimum point as long as possible. Unless the profitability and resource utilization of the business operation, especially the assignment of workers or machines to corresponding jobs or activity, adversely affect efficiency, productivity, and long-run market competition.

5 | Conclusion

In this article, fundamental concepts that scholarly underscored and a quantitative tool of management science were introduced. This is through aligning with the potential application areas (real case) of the AP which is the special structure of the transportation problem. As an exemplar, current real-case resource allocation activities' problems are defined by aligning their parameters with generic inputs of the HAA that LPP governs. The methods of conversion of unbalanced linear programming matrix applied to procedurally apply the selected algorithm (Hungarian method) to achieve the aim of the study. Following the minimization objective function of the model, an optimal cost suggested less than or equal to 18 per unit, while the maximization objective function indicated 100 values or profit. An author indicated that to be profitable and manage unwanted expenses in daily operational management, the current case firm and the other relevant operational areas are highly expected to pursue a factual decision-making approach.

Generally, this article also noticed that, the roles of the quantitative management approach in resource assignment activities (linear programming) in industrial practical activities for effective resource utilization and long-run competitiveness.

The author of this article would like to forward the following activities predominantly for the current case firm and universally for all industrial practitioners in resource allocation operational management processes:

I. Operational and concerned bodies of this case firm should pursue a quantitative decision-making approach rather than follow imaginary conclusions.

II. Overall, costs incurred due to allocating resources in operational activities need to be monitored and should not be more than 18 per unit.

III. Before assigning resources in scheduling operations, practitioners need to have sufficient parameters and effects of one variable on the others.

IV. Managers should pursue a quantitative decision-making approach rather than follow imaginary conclusions

V. Monitoring and tracking the progress of the scheduled operational timeline along with resource utilization should be quantitatively reported, and corrective measurements should be timely taken.

Through these and other relevant tools and quantitative management approaches, the case company and other similar firms can optimize their resource utilization and reduce opportunity losses, which can adversely affect their productivity.

Declaration

The author would like to declare hereby that the work being presented in this article entitled "Optimizing Resource Utilization Using an Assignment Model in Case of Metal Workshop" is an original work of my investigation, which has not been presented for publications, and all the resources, materials used for this study have been properly acknowledged.

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Conflicts of Interest

An author has seen and agrees with the manuscript's contents, and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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